Refutation of the quaternion based on its implication truth table

Abstract: We evaluate the standard definition of the quaternion as four equations to produce its multiplication table. We then derive the implication table for the quaternion which is *not* tautologous, and hence refutes the quaternion. This result forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with `T`autology as the designated proof value, **`F`** as contradiction, `N` as truthity (non-contingency), and `C` as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ , · , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ∃!, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) `T` as tautology, ⊤, ordinal 3; (z@z) **`F`** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) `N` as non-contingency, Δ, ordinal 1; (%z<#z) `C` as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

A standard definition of the quaternion is the four multiplication expressions:

ij=k (1.1), (2.1)
jk=i (3.1), (4.1)
ki=j (5.1), (6.1)
ijk=-1 (7.1), (8.1)

Eq. 7.1, 8.1 is expanded by substitution as:

(4.1)*(6.1)*(2.1)=(8.1) (9.1)

LET p, q, r, s: i, j, k, s.

((p&q)&r)=~(%s>#s) ; `NNNC NNNN NNNC NNNN` (9.2)

The multiplication table is:

(10.1)

x	1	i	j	k
1	1	i	j	k
i	i	-1	k	-j
j	j	-k	-1	i
k	k	j	-i	-1

Row 4: (((r&(%s>#s))=r)+((r&p)=q))+(((r&q)=~p)+((r&r)=~(%s>#s))) ;
`TTTT TTTT TTTT TTTT` (10.2)

Remark 10.2: The multiplication table is confirmed as tautologous because rows (and columns) produce the same result of tautology as in Eq. 10.2 for Row 4.

The implication table is produced by assigned values as (A>B), or based on (A>B)=~(A&~B) ie the Imply connective defined in terms of Not And, with repeating rows shown. (Similarly, the And connective can be defined in terms of the Not Imply connective as (A&B)=~(A>~B).)

p=(s=s) ;	i	FTFT FTFT FTFT FTFT
q=(s=s) ;	j	FFTT FFTT FFTT FFTT
r=(s=s) ;	k	FFFF TTTT FFFF TTTT
s=(s=s) ;	fyi	FFFF FFFF TTTT TTTT
(%s>#s) ;	1	NNNN NNNN NNNN NNNN
~p=(s=s) ;	~i	TFTF TFTF TFTF TFTF, etc
~(%s>#s) ;	-1	CCCC CCCC CCCC CCCC

→	1	i	j	k
1	TTTT TTTT	CTCT CTCT	CCTT CCTT	CCCC TTTT
i	TNTN TNTN	TTTT TTTT	T**F**TT T**F**TT	T**F**T**F** TTTT
j	TTNN TTNN	TT**F**T TTFT	TTTT TTTT	TT**FF** TTTT
k	TTTT NNNN	TTTT **F**T**F**T	TTTT **FF**TT	TTTT TTTT

(11.1)

Row 1: (((%s>#s)&(%s>#s))+ … ; NNNN NNNN … = %s>#s [1]
Row 2: ((p&(%s>#s))+(p&p))+((p&q)+(p&r)) ; FTFT FTFT … = p [i]
Row 3: ((q&(%s>#s))+(q&p))+((q&q)+(q&r)) ; FFTT FFTT … = q [j]
Row 4: ((r&(%s>#s))+(r&p))+((r&q)+(r&r)) ; FFFF TTTT … = r [k]

Column 1: (((%s>#s)&(%s>#s))+(p&(%s>#s)))+((q&(%s>#s))+(r&(%s>#s))) ;
NNNN NNNN … = %s>#s [1]
…
Column 4: ((r&(%s>#s))+(r&p))+((r&q)+(r&r)) ; FFFF TTTT … = r [k]

(11.2)

Remark 11.2: Because in Eq. 11.2 the row and column result is the respective table index value, the quaternion implication table of 11.1 is *not* tautologous. (It should be all T's as in 10.2.) Hence the quaternion is refuted on the basis of the truth table for its imply connective.