

Abstract: We evaluate these topics using the Meth8/VŁ4 modal logic model checker:

Refutation of axiom of choice in 1 operator or quantifier, 2 variables, and 4 connectives

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, F as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬; + Or, ∨, ∪, ⊔; - Not Or; & And, ∧, ∩, ∏, ·, °, ⊗; \ Not And;
 > Imply, greater than, →, ⇒, ⇨, >, ⊃, ⇒; < Not Imply, less than, ∈, <, ⊂, ≠, ≠, ←, ≲;
 = Equivalent, ≡, :=, ⇔, ↔, ≐, ≈, ≃;
 @ Not Equivalent, ≠, ⊕;
 % possibility, for one or some, ∃, ∃!, ∃, M; # necessity, for every or all, ∀, □, L;
 (z=z) T as tautology, T, ordinal 3; (z@z) F as contradiction, Ø, Null, ⊥, zero;
 (%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
 ~(y < x) (x ≤ y), (x ⊆ y), (x ⊇ y); (A=B) (A~B).
 Note for clarity, we usually distribute quantifiers onto each designated variable.

We cast the axiom of choice in these words:

If possibly filled bins imply selection or no selection, and
 If not possibly filled bins imply no selection. (1.1)

LET p, q: filled bins, selection.

(%p>(q+~q))&(~%p>~q); TTCT TTCT TTCT TTCT (1.2)

Remark 1.2: Eq. 1.1 can be weakened with modal necessity or universal quantification. (2.1)

(#p>(q+~q))&(~#p>~q); TTFN TTFN TTFN TTFN (2.2)

Eq. 1.2 or 2.2 as rendered is *not* tautologous, hence refuting the axiom of choice in one modal operator or one quantifier, two variables, and four connectives.