

Refutation of Cantor's continuum by his own axiom of infinity, invalidating aleph-zero ($\aleph_0$)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ∃!, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N̲ as non-contingency, Δ, ordinal 1; (%z<#z) C̲ as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

From: Leon, A. (2021). "Infinity put to the text". vixra.org\2102.0121v1.pdf

P46 The Axiom of Infinity will be now introduced through three stages of an increasing abstraction. The less formal version of the Axiom of Infinity goes as follows:

$$\textit{There exists an infinite denumerable set} \tag{3}$$

where denumerable (or enumerable) means that it can be put into a one to one correspondence with the set $\mathbb{N} = \{1, 2, 3 \dots\}$ of the natural numbers, and infinite stands for the actual infinity: the elements of that set exist all at once, as a complete totality. Two sets that can be put into a one to one correspondence (said equipotents or equinumerous sets) either both are finite or both are infinite. The second more abstract form of the Axiom of Infinity is the following one:

$$\exists N(0 \in N \land \forall x \in N(s(x) \in N)) \tag{4}$$

that reads: there exist a set N [symbols: $\exists N$] such that 0 belongs to N [symbols: $0 \in N$] and for all element x in N [symbols: $\land \forall x \in N$] the successor of x, denoted by $s(x)$, also belongs to N [symbols: $s(x) \in N$]. In arithmetical terms we could write:

$$s(0) = 1;\ s(1) = 2;\ s(2) = 3; \dots \tag{5}$$

Therefore, the Axiom of Infinity establishes the existence of a set comparable to the set of the natural numbers. And the third still more abstract form of the Axiom of Infinity is:

$$\exists N(\emptyset \in N \land \forall x \in N(x \cup \{x\} \in N)) \tag{6}$$

that reads: there exists a set N such that $\emptyset$ (the empty set) belongs to N and for all elements x in N, the element $x \cup \{x\}$ (x and a set whose unique element is x) also belongs to N. Though the existence of an actual infinity can be inferred from both (4) and (6), it would have been better a more explicit declaration that the infinity implicated in the axiom is the actual infinity.

(46.3.1 - 46.6.1)

LET p, q, r, s: x, N, r, s.

In arithmetical terms, Eq. 46.5.1 reduces to the sequence of [s(0)=1; s(1)=2; s(2)=3; ...] as all less than N. However, N is not verifiable as countable in a lifetime, so it makes better sense to define the opposite of at least one P as zero (or one), whereby the sequence is always greater than or equal to the respective P.

The state with at least one P as zero has the sequence as all greater than or equal to zero, with antecedent, consequent, and result as respectively: (46.5.1.1.1)

((((#q+(s@s))=(%s>#s))+((#q+(%s>#s))=(%s<#s)))+((#q+(%s<#s))=(s=s))) = (s=s) ;
CCTT CCTT CCTT CCTT (46.5.1.1.2)

%(s@s) = (s=s) ; CCCC CCCC CCCC CCCC (46.5.1.1.3)

~(%(s@s)>((((#q+(s@s))=(%s>#s))+((#q+(%s>#s))=(%s<#s)))+((#q+(%s<#s))
=(s=s)))) = (s=s) ; **FFFF FFFF FFFF FFFF** (46.5.1.1.4)

Remark 46.5.1.1.4: Eq. 46.5.1.1.4 is *not* tautologous, and in fact contrary, to refute the axiom of infinity counted from zero as a theorem, denying the continuum hypothesis.

The problem is that zero is not a countable natural number but rather a dividing point. This fact seemed to elude Cantor, Hilbert, Gödel, and Cohen et al. Hence the axiom of infinity is based on the smallest countable natural number which is one, below.

The state with at least one P as one has the sequence as all greater than or equal to one, but *without* zero as an index, with antecedent, consequent, and result as respectively: (46.5.1.2.1)

((#q+(%s>#s))=(%s<#s))+((#q+(%s<#s))=(s=s)) ;
CCTT CCTT CCTT CCTT (46.5.1.2.2)

%(%s>#s) = (s=s) ; TTTT TTTT TTTT TTTT (46.5.1.2.3)

~(%(%s>#s)>(((#q+(%s>#s))=(%s<#s))+((#q+(%s<#s))=(s=s)))) = (s=s) ;
NN**FF** NN**FF** NN**FF** NN**FF** (46.5.1.2.4)

Remark 46.5.1.2.4: Eq. 46.5.1.2.4 is *not* tautologous, to refute the axiom of infinity counted from one as a theorem, denying the continuum hypothesis.

(For 46.5.1.2.1, it turns out that if zero is an index for the first s(0), then truth table results for Eqs. 46.5.1.2.2 - 46.5.1.2.4 still remain the same.)

In other words, the continuum hypothesis is refuted with classical logic as based on natural numbers.

This further refutes the Gödel and Cohen efforts claiming to show the continuum hypothesis is effectively undecidable inside or outside of set theory. What follows is aleph-zero ($\aleph_0$) as moot, denying contingent conjectures in set theory.