

Refutation of bijective function as basis of cardinals, Cantor set theory, and modern foundation

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕;
% possibility, for one or some, ∃, ∃!, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

Note: This evaluation was sparked by the logicandreligion.com/webinars of 10/14/2021, "What are the arguments for atheism?", with Piergiorgio Odifreddi (University of Turin, Italy), Jan Woleński (Jagiellonian University, Poland), and Stanisław Krajewski (University of Warsaw, Poland). The participants and most listeners had one thing in common: assumption of Georg Cantor's cardinal number and set theory as the foundation of modern mathematics. Cardinality is commonly defined through the bijective function below.

From: en.wikipedia.org/wiki/Bijection

For a pairing between *X* and *Y* (where *Y* need not be different from *X*) to be a bijection, four properties must hold: (1.0.1)

1. Each element of *X* must be paired with at least one element of *Y*;
2. No element of *X* may be paired with more than one element of *Y*;
3. Each element of *Y* must be paired with at least one element of *X*, and
4. No element of *Y* may be paired with more than one element of *X*.

(1.1.1 - 1.5.1)

LET p, q: X, Y.

(p=q)+~(p=q) ; TTTT TTTT TTTT TTTT (1.0.2)
#p>%q ; TCTT TCTT TCTT TCTT (1.1.2)
~p>#q ; **F**TNT **F**TNT **F**TNT **F**TNT (1.2.2)
#q>%p ; TTCT TTCT TTCT TTCT (1.3.2)
~q>#p ; **F**NTT **F**NTT **F**NTT **F**NTT (1.4.2)
((#p>%q)&(~p>#q))&((#q>%p)&(~q>#p)) ;
FFFT **FFF**T **FFF**T **FFF**T (1.5.2)
(#p>%q)&(~p>#q) ; **F**CNT **F**CNT **F**CNT **F**CNT (1.5.3)
(#q>%p)&(~q>#p) ; **F**NCT **F**NCT **F**NCT **F**NCT (1.5.4)

Remark 1.5.2: Eq. 1.5.2 is *not* tautologous, to refute the bijective function as claimed, denying the basis of cardinals, Cantor set theory, and foundation of modern mathematics. (Eqs. 1.5.3 and 1.5.4 are the antecedent and consequent of 1.5.2.)