

Refutation of Peirce's abduction and induction, and confirmation of deduction

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ , | ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And, ↑ ;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ∃!, ◊, M ; # necessity, for every or all, ∀, □, L ;
(z=z) T as tautology, ⊤, ordinal 3 ; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero ;
(%z>#z) N as non-contingency, Δ, ordinal 1 ; (%z<#z) C as contingency, ∇, ordinal 2 ;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

11.5. Refutation of Peirce's abduction and induction, and confirmation of deduction

[This was taken from a 2019 dissertation draft for reproduction here after renewed interest in rules of inference.]

From: iep.utm.edu/peir-log/

C.S. Peirce originally defined the three forms of inference in logic as:

Abduction: (Q is S) and (Q is P) imply (S is P) (11.5.1.1.1)

LET p, q, s: P, Q, S.

((q=s)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT (11.5.1.1.2)

Induction: (S is Q) and (P is Q) imply (S is P) (11.5.2.1.1)

((s=q)&(p=q))>(s=p) ; TTTT TTTT TTTT TTTT (11.5.2.1.2)

Deduction: (S is Q) and (Q is P) imply (S is P) (11.5.3.1.1)

((s=q)&(q=p))>(s=p) ; TTTT TTTT TTTT TTTT (11.5.3.1.2)

Peirce described Eqs. 11.5.1 - 11.5.3 as inversions of the same.

Remark 11.5.1.1.1: If the word "is" is taken to mean the word "implies" then the connective = is replaced with the connective > below.

Abduction: (Q implies S) and (Q implies P) imply (S implies P) (11.5.1.2.1)

((q>s)&(q>p))>(s>p) ; TTTT TTTT **F**TTT **F**TTT (11.5.1.2.2)

Induction: (S implies Q) and (P implies Q) imply (S implies P) (11.5.2.2.1)

((s>q)&(p>q))>(s>p) ; `TTTT TTTT TT`**`F`**`T TT`**`F`**`T` (11.5.2.2.2)

Deduction: (S implies Q) and (Q implies P) imply (S implies P) (11.5.3.2.1)

((s>q)&(q>p))>(s>p) ; `TTTT TTTT TTTT TTTT` (11.5.3.2.2)

Remark 11.5.1.2.2-11.5.2.2.2: Eqs. 11.5.1.2.2 - 11.5.2.2.2 as rendered for abduction and induction are *not* tautologous, but Eq. 11.5.3.2.2 is tautologous. This means that abduction and induction are not inversions of deduction, leaving deduction as the only form of tautologous inference in logic.