

Resuscitation of Peirce's bag of white beans as a syllogistic to prove only deduction

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ , | ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘ , ⊗ ; \ Not And, ↑ ;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ∃!, ◊, M ; # necessity, for every or all, ∀, □, L ;
(z=z) T as tautology, ⊤, ordinal 3 ; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero ;
(%z>#z) N as non-contingency, Δ, ordinal 1 ; (%z<#z) C as contingency, ∇, ordinal 2 ;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); ~(x < y) (x ≥ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

From: Clough, R.B. (no date). abductive, inductive and deductive syllogisms of Peirce.odt.
academia.edu/30330155/abductive_inductive_and_deductive_syllogisms_of_Peirce_odt
[*Note:* this title is nowhere listed in the publications of that author.]

These are taken from "Peirce's Cosmology" by Peter Turley, Philosophical library (1977), pp13-14.

1. Abductive syllogism (tentative conclusion):

All the beans in this bag are white. [1]
These beans are white. [2]
These beans are from this bag. [3]

2. Inductive syllogism:

These beans are from this bag. [3]
These beans are white. [2]
All the beans in this bag are white. [1]

3. Deductive syllogism:

All the beans in this bag are white. [1]
These beans are from this bag. [3]
These beans are white. [2]

Note that 1 and 2 are simply a reversal of order, ie abduction is simply induction performed in reverse order. (0.0)

Remark 0.0: It turns out that the above proves induction only, and the truth table values for abduction and induction are disparate.

From logical truth table value results, induction has the same result for 3&2>1 as for 2&3>1, which means abduction with the same result for 1&2>3 as for 2&1>3 is not necessarily induction performed in reverse order to abduction. Hence order of terms can be misleading.

From: Beckwith, A. (2018). C.S. Peirce and abduction inference. JCCC Honors Journal. 10:1:2. scholarspace.jccc.edu/honors_journal/vol10/iss1/2

Deduction

However, to clarify the argument better to fit the Barbara pattern, the statements are rewritten as follows:

LET p, q, s: beans, bag, white.

[1] Rule: All these beans from this bag are white. (1.1.1)

(%q>#p)>s ; CCTC CCTC TTTT TTTT (1.1.2)

[2] Case: All these beans are beans from this bag. (1.2.1)

#(%q>%p) = (s=s) ; NN**F**N NN**F**N NN**F**N NN**F**N (1.2.2)

[3] Result: All these beans are white. (1.3.1)

#(%p>s) = (s=s) ; N**F**N**F** N**F**N**F** NNNN NNNN (1.3.2)

Rule and Case imply Result: (1&2)>3: (1.4.1)

(((%q>#p)>s)&#(%q>%p))>#(%p>s) ;
TTTT TTTT TTTT TTTT (1.4.2)

Induction

[3] Result: All these beans are white.
[2] Case: All these beans are beans from this bag.
[1] Rule: All these beans from this bag are white.

Result and Case imply Rule: (3&2)>1: (1.5.1)

(#(%p>s)&#(%q>%p))>((%q>#p)>s) ;
CTTT CTTT TTTT TTTT (1.5.2)

Abduction

[1] Rule: All these beans from this bag are white.
[3] Result: All these beans are white.
[2] Case: All these beans are beans from this bag.

Rule and Result imply Case: (1&3)>2: (1.6.1)

(((%q>#p)>s)&#(%p>s))>#(%q>%p) ;
TTCT TTCT TTCT TTCT (1.6.2)

Remark 1.4.2: Eq. 1.4.2 is tautologous to confirm deduction, but 1.5.2 for induction and 1.6.2 for abduction are not tautologous, to refute the bag of white beans conjecture as proof for the *three* rules of inference.

We next evaluate the three forms of inference in the same three-fold method of ((x)&(y))>(z) by arbitrarily assigning alphabetically abduction, deduction, and induction to respectively case, result, and rule.

Abduction and induction imply deduction (1.7.1)

(((((%q>#p)>s)&#(%p>s))>#(%q>%p))&((#(%p>s)&#(%q>%p))>((%q>#p)>s)))>
((((%q>#p)>s)&#(%q>%p))>#(%p>s)) ;

`TTTT TTTT TTTT TTTT` (1.7.2)

Abduction and deduction imply induction (1.8.1)

(((((%q>#p)>s)&#(%p>s))>#(%q>%p))&((((%q>#p)>s)&#(%q>%p))>#(%p>s)))>
((#(%p>s)&#(%q>%p))>((%q>#p)>s)) ;

`CTTT CTTT TTTT TTTT` (1.8.2)

Induction and deduction imply abduction (1.9.1)

(((#(%p>s)&#(%q>%p))>((%q>#p)>s))&((((%q>#p)>s)&#(%q>%p))>#(%p>s)))>
((((%q>#p)>s)&#(%p>s))>#(%q>%p)) ;

`TTCT TTCT TTCT TTCT` (1.9.2)

Remark 1.7.2: Eq. 1.7.2 is tautologous to confirm that abduction and induction imply deduction, but no other combinations in the three-fold format are confirmed.

This further replicates and confirms our previous results using P, Q, S from

ersatz-systems.com/Digest%202022.07.16.01.pdf .