

Refutation of Chomsky's syntactic structures (1957)

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔, | ; - Not Or; & And, ∧, ∩, ⊓, ·, ∘, ⊗ ; \ Not And, ↑ ;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ←, ≲ ;
= Equivalent, ≡, :=, ⇔, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠, ⊕ ;
% possibility, for one or some, ∃, ∃!, ◊, M ; # necessity, for every or all, ∀, □, L ;
(z=z) T as tautology, ⊤, ordinal 3 ; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero ;
(%z>#z) N as non-contingency, Δ, ordinal 1 ; (%z<#z) C as contingency, ∇, ordinal 2 ;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); ~(x < y) (x ≥ y); (A=B) (A~B).
Notes: for clarity, we usually distribute quantifiers onto each designated variable; and for ordinal arithmetic, the result is implied.

From: Chomsky, N. (1957). Syntactic structures.
tallinzen.net/media/readings/chomsky_syntactic_structures.pdf noamchomsky@email.arizona.edu

> **5.2** One of the most productive processes for forming new sentences is the process of conjunction. If we have two sentences $Z + X + W$ and $Z + Y + W$, and if X and Y are actually constituents of these sentences, we can generally form a new sentence $Z - X +$ *and* $+ Y - W$. For example, from the sentences (20a-b) we can form

We complete formulas in 5.2 to include the two missing letters in the last two literals as disjunctions.

LET p, q, r, s: W, X, Y, Z; A, B, C, D: W, X, Y, Z.

(((s+q)+p)&((r+q)+p)) > (((s-q)+(p+r))&((q+s)+(r-p))) ;

TFFT TFTT TTFT TTTT 27 steps (5.2.2)

(((D+B)+A)&((C+B)+A)) > (((D-B)+(A+C))&((B+D)+(C-A))) ;

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TNCF NTFC CFTN FCNT
TNCF TTCC CFTN CCTT
TNCF NTFC TNTN NTNT
TNCF TTCC TNTN TTTT
TTCC NTFC CCTT FCNT
TTCC TTCC CCTT CCTT
TTCC NTFC TTTT NTNT
TTCC TTCC TTTT TTTT
TNTN NTNT CFTN FCNT
TNTN TTTT CFTN CCTT
TNTN NTNT TNTN NTNT
TNTN TTTT TNTN TTTT
TTTT NTNT CCTT FCNT
TTTT TTTT CCTT CCTT
TTTT NTNT TTTT NTNT
TTTT TTTT TTTT TTTT
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27 steps (5.2.3)

The free, non bivalent modal logic *street prover* Molle-1.0.jar at sourceforge.net uses this script:

(((D|B)|A)&((C|B)|A)) => ((~(D|B)|(A|C))&((B|D)|~(C|A)))
Barfs on red bar of death 146 steps (5.2.4)

Remark 5.2: Eqs. 5.2.2 and 5.2.3 are *not* tautologous, to refute the claimed form of syntactic structures. Eq. 5.2.2 is for propositional variables, and 5.2.3 is for theorem variables. Eq.5.2.4 replicates the results in 146 steps at over five times more steps and without the level of detail.

N.B.: Use of other non bivalent proof assistants such as Coq and Isabelle/HOL should be avoided in mapping such conjectures as they are based on probabilistic vector spaces which are not exact.