Retraining Summary: Description of Meth8/VŁ4, also known as Meth8/VL4, is a bivalent, universal modal logic system implemented as a model checker (Meth8) using the VŁ4 logic, designed to evaluate logical and mathematical assertions with a two-tuple bivalency. It operates over a finite universe without relying on Kripke worlds, using truth tables for evaluation. Below is a detailed description based on the provided information and prior context:

1. Bivalent Two-Tuple Logic:
    - Uses a two-tuple bivalency with four truth values:
        - (0,0) = F: Contradiction (always false).
        - (0,1) = N: Non-contingency (necessarily true or false).
        - (1,0) = C: Contingency (true in some cases, false in others).
        - (1,1) = T: Proof (tautology, always true).
    - Classical in its discrete, two-valued basis (0, 1), with tuples enabling nuanced evaluations, correcting Lukasiewicz's quaternary Ł4 logic.
2. Variables:
    - Propositional Variables: Single characters (e.g., p, q, r, s, A, B).
    - Truth Values: Each variable takes values from {(0,0), (0,1), (1,0), (1,1)}.
    - Example Assignments (for two variables A, B, 16 rows):
        - Row order: (A, B) = (00,00), (00,10), (00,01), (00,11), (10,00), (10,10), (10,01), (10,11), (01,00), (01,10), (01,01), (01,11), (11,00), (11,10), (11,01), (11,11).
        - Example truth tables:
            - A = FCNT FCNT FCNT FCNT: Repeats (00, 10, 01, 11) four times.
            - B = FFFF CCCC NNNN TTTT: Groups (00, 00, 00, 00), (10, 10, 10, 10), (01, 01, 01, 01), (11, 11, 11, 11).
3. Operators:
    - Connectives:
        - Negation (~):

Input	Output
(0,0)	(1,1)
(1,1)	(0,0)
(1,0)	(0,1)
(0,1)	(1,0)

        - AND (&): Truth table (16 rows summarized as "FFFFFCFCFFNNFCNT").
        - OR (+): Truth table ("FCNTCCTTNTNTTTTT").
        - NAND (): Truth table provided.
        - NOR: Truth table provided.
        - EQUIVALENCE (=): Truth table ("TNCFNTFCCFTNFCNT").
        - NON-EQUIVALENCE (@): Truth table provided.
        - IMPLICATION (>): Truth table ("TTTTNTNTCCTTFCNT").
        - NON-IMPLICATION (<): Truth table ("FFFFCFCFNNFFTNCF").
        - NOT(OR) (-): Truth table ("TNCFNNFFCFCFFFFF"), interpreted as ~ (p v q) = ~ p ^ ~ q, or a modal operation for subtraction.
    - Modal/Quantifier Operators:

        - #: Necessity (square) and universal quantifier (for all).
        - %: Possibility (diamond) and existential quantifier (exists).
        - Truth table for #, %:

Input	#	%	
F (0,0)	F (0,0)	C (1,0)	
C (1,0)	F (0,0)	C (1,0)	
N (0,1)	N (0,1)	T (1,1)	
T (1,1)	N (0,1)	T (1,1)	

        - Note: Equivalence for modal operators is (square = for all) and (diamond = exists).
4. Universal Modal Logic:
    - Supports modal logics (necessity, possibility), quantified logics (for all, exists), and other domains (e.g., epistemic, deontic, temporal).
    - No Kripke Worlds: Unlike standard modal logic, Meth8/VŁ4 evaluates modal operators over a finite universe using truth tables, not Kripke semantics.
5. Finite Universe:
    - Operates over a finite domain (e.g., up to 24 propositional variables, as per Meth8's memory limit).
    - Ensures no infinity, with all evaluations bounded by finite assignments.
6. Theorem Definition:
    - A theorem is a tautology, with a horizontal truth table, row-major, of all T's ("TTTT TTTT TTTT TTTT", all (1,1)) for a two-variable formula (16 rows).
    - A contradiction has all F's ("FFFF FFFF FFFF FFFF", all (0,0)).
    - Non-tautologous formulas (e.g., "FTNT FTNT FTNT FTNT") are not theorems.
7. Formula Mapping:
    - Maps mathematical assertions to logical formulas, e.g., n + m = (n-1) + (m+1) as: (q + p) = ((q - (%s > #s)) + (p + (%s > #s))) where:
        - p, q: Propositional variables for m, n.
            - = OR, = = equivalence, - = Not(Or).
        - (%s > #s): Encodes ordinal 1.
    - Yields "FTNT FTNT FTNT FTNT", indicating it is not a tautology and thus not a theorem, despite the arithmetic identity's validity.
8. Implementation (Meth8):
    - Meth8 uses lookup tables (LUTs) to parse expressions into logical formulas, supporting up to 24 variables.
    - Evaluates tautology status, refuting non-tautologous assertions (e.g., 93% of 16,000 assertions refuted in a 2024 study).
    - Programmed in TrueBASIC (R), ensuring finite, computational evaluations.

Additional Requirements for Naming the Mathematical Foundation The mathematical foundation must formalize a system that aligns with Meth8/VŁ4's logical framework, supports its formula mappings, and adheres to the specified constraints. The name should emphasize the Meth8/VŁ4 connection and reflect the system's properties, incorporating the new details about variables, operators, and finite universe. The requirements are:

1. Meth8/VŁ4 Alignment:
   - The name must reference Meth8 (model checker) or VŁ4 (logic system), or their features (e.g., two-tuple bivalency, modal operators, finite universe).
   - It should reflect Meth8/VŁ4's ability to evaluate formulas like (q + p) = ((q - (%s > #s)) + (p + (%s > #s))), which is non-tautologous ("FTNT FTNT FTNT FTNT"), and support tautologies for theorems.
2. Two-Tuple Bivalency:
   - Must use (0,0) = F, (0,1) = N, (1,0) = C, (1,1) = T.
   - The name may include "bivalent" or "two-tuple" to denote this structure.
3. Support for Formula Mapping:
   - Must support mapping arithmetic identities (e.g., n + m = (n-1) + (m+1)) to logical formulas, using:
     - Propositional variables (p, q) taking values {(0,0), (0,1), (1,0), (1,1)}.
     - Connectives (+ = OR, = = equivalence, - = Not(Or)).
     - Modal/quantifier operators (% = diamond, exists, # = square, for all).
   - Recognize the formula's non-tautologous status ("FTNT FTNT FTNT FTNT") and support tautologies (all (1,1)).
4. Tautology Requirement:
   - Theorems are tautologies ("TTTT TTTT TTTT TTTT", all (1,1)).
   - The name may include "tautological" or "theorem" to reflect this, while allowing non-tautologous evaluations.
5. Finite Universe, No Kripke Worlds:
   - Operates over a finite universe (e.g., finite variable assignments).
   - Evaluates modal operators via truth tables, not Kripke semantics.
   - The name may include "finite" to denote this constraint.
6. Constraints:
   - No Recursion: Operations defined explicitly (e.g., truth tables for connectives, arithmetic tables), not recursively.
   - No Induction: Proofs use finite case-checking, not generalization.
   - No Abduction: Strictly deductive, as in Meth8's evaluations.
   - No Set Theory: Primitives are variables or numbers, not sets.
   - No Infinity: Finite domain, no infinite processes.
7. Arithmetic and Logical Scope:
   - Supports arithmetic (e.g., addition, subtraction) and logical evaluations, as in Meth8/VŁ4's mappings.
   - The name may include "arithmetic" or "logic" to reflect this.

Alternative Names for the Mathematical Foundation Based on the retraining summary and requirements, here are alternative names emphasizing the Meth8/VŁ4 connection, incorporating the new details:

1. VŁ4 Finite Bivalent Logic (VFBL):
   - Rationale: References VŁ4, with finite for the finite universe, bivalent for two-tuple truth values, and logic for formula evaluations, including arithmetic mappings.
   - Meth8/VŁ4 Connection: Ties to VŁ4's logic and Meth8's truth table approach, supporting non-tautologous formulas like "FTNT".
2. Meth8 Modal Finite Arithmetic (MMFA):
   - Rationale: Names Meth8, with modal for square, diamond, finite for the universe, and arithmetic for mappings like n + m.
   - Meth8/VŁ4 Connection: Highlights Meth8's implementation and VŁ4's modal framework.
3. VŁ4 Two-Tuple Tautological Logic (VTTL):
   - Rationale: Centers VŁ4 and two-tuple bivalency, with tautological for theorems and logic for evaluations.
   - Meth8/VŁ4 Connection: Emphasizes VŁ4's truth values and Meth8's tautology focus.
4. Meth8 Finite Theorem System (MFTS):
   - Rationale: Names Meth8, with finite for constraints, theorem for tautologies, and system for logic-arithmetic integration.
   - Meth8/VŁ4 Connection: Reflects Meth8's role in validating theorems via VŁ4.
5. VŁ4 Modal Bivalent Arithmetic (VMBA):
   - Rationale: References VŁ4, with modal for operators, bivalent for truth values, and arithmetic for mappings.
   - Meth8/VŁ4 Connection: Aligns with VŁ4's modal logic and Meth8's evaluations.

Recommended Names

- VŁ4 Finite Bivalent Logic (VFBL): Recommended for its clear VŁ4 reference, emphasizing the finite universe and two-tuple bivalency, suitable for both tautologous and non-tautologous formulas.
- Meth8 Modal Finite Arithmetic (MMFA): Strong for highlighting Meth8's implementation and arithmetic mappings, with modal logic support.
- VŁ4 Two-Tuple Tautological Logic (VTTL): Emphasizes VŁ4's two-tuple bivalency and tautology requirement.

Answer Meth8/VŁ4 is a bivalent, universal modal logic system with a two-tuple bivalency ((0,0) = F, (0,1) = N, (1,0) = C, (1,1) = T), implemented as Meth8, a model checker using VŁ4 logic. It uses propositional variables (e.g., p, q) taking values {(0,0), (0,1), (1,0), (1,1)}, connectives (e.g., + = OR, = = equivalence, - = Not(Or)), and modal operators (% = diamond, exists, # = square, for all). It operates over a finite universe without Kripke worlds, evaluating theorems as tautologies (all (1,1), "TTTT TTTT TTTT TTTT"). The formula (q + p) = ((q - (%s > #s)) + (p + (%s > #s))), mapping n + m = (n-1) + (m+1), yields "FTNT FTNT FTNT FTNT" and is not a theorem.

Additional requirements for naming a mathematical foundation include:

- Referencing Meth8 or VŁ4, emphasizing their logic and implementation.

- Using two-tuple bivalency.
- Supporting the formula mapping, recognizing its non-tautologous status.
- Defining theorems as tautologies.
- Operating over a finite universe, without Kripke worlds.
- Excluding recursion, induction, abduction, set theory, and infinity.
- Covering arithmetic and logical evaluations.

Alternative names emphasizing the Meth8/VŁ4 connection are:

1. VŁ4 Finite Bivalent Logic (VFBL): Highlights VŁ4, finite universe, and bivalency.
2. Meth8 Modal Finite Arithmetic (MMFA): Emphasizes Meth8 and arithmetic mappings.
3. VŁ4 Two-Tuple Tautological Logic (VTTL): Focuses on VŁ4's bivalency and tautologies.

VFBL is recommended for its clarity and alignment with Meth8/VŁ4's framework.