Refuting the Axiom of Infinity in quad-valent universal modal logic system meth8/vł4: Implications for finitist Trinitarian logic

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Abstract
This paper integrates Trinitarian logic, formalized within the Meth8/VŁ4 bivalent modal logic system, to evaluate and refute the Axiom of Infinity—a foundational principle of Zermelo-Fraenkel set theory (ZFC) that asserts the existence of an infinite set. Employing Meth8/VŁ4's quad-valent truth values (F, N, C, T) and modal operators, we demonstrate that the Axiom of Infinity is non-tautologous in a finite universe, yielding results such as TTTC CCCT TTTC CCCT. This refutation aligns with Meth8/VŁ4's finitist framework, rejecting infinite set theory while supporting a coherent finitist Trinitarian theology. By modeling divine unity (p = Father, q = Son, r = Spirit) and human-divine relations (s = man) as tautologies, this work bridges mathematical logic and analytical theology, offering a formal tool for evaluating foundational axioms.

Keywords: Axiom of Infinity, finitist logic, Meth8/VŁ4, modal logic, non-tautologous, quad-valent logic, set theory, Trinitarian logic, truth table, Zermelo-Fraenkel

1. Introduction

The Meth8/VŁ4 system is a quad-valent, bivalent modal logic framework designed for deductive model checking, utilizing four truth values: **F** (contradiction, (0,0)), **N** (non-contingent, (0,1)), **C** (contingent, (1,0)), and **T** (tautology, (1,1)). Only **T** validates theorems, requiring all **T**'s in truth tables. Unlike systems relying on Kripke semantics, Meth8/VŁ4 operates within a finite universe, inherently rejecting infinite sets. This paper applies Meth8/VŁ4 to evaluate the Axiom of Infinity, a ZFC axiom positing an infinite set (e.g., {Ø, {Ø}, {{Ø}}, …}), and extends this analysis through Trinitarian logic—a formalization of Christian theology rooted in scripture (e.g., Mt 28:19, Jn 1:1) and creeds (Nicene, Chalcedonian).

Trinitarian logic unifies 27 theological topics—such as divine causation, incarnation, and anthropology—as tautologous formulas (TTTT TTTT TTTT TTTT), using variables **p** (Father), **q** (Son), **r** (Spirit), and **s** (man), alongside others like **m** (Mary), **a** (angels), and **d** (demons). Divine unity is modeled as **p = q = r**, while **s** reflects human contingency (e.g., **s = F** for sin). By applying this framework to the Axiom of Infinity, we refute its validity in Meth8/VŁ4, supporting finitist theology and challenging infinite mathematical constructs.

2. Meth8/VŁ4 Framework

Meth8/VŁ4 is a quad-valent logic system in classical logic.

Truth Values:

- **False (F, 0,0)**: Contradiction (always false).
- **Contingent (C, 1,0)**: True in some cases, false in others.
- **Non-contingent (N, 0,1)**: Necessarily true/false, not tautologous.
- **True (T, 1,1)**: Tautologous (always true, designated proof value).

Operators (4x4 truth tables, row-major, FCNT order):

- **Negation (~)**: F→T, T→F, C→N, N→C.
- **Conjunction (&)**: FFFF FCFC FFNN FCNT.
- **Implication (>)**: TTTT NTNT CCTT FCNT.
- **Equivalence (=)**: TNCF NTFC CFTN FCNT.
- **Non-equivalence (@)**: FCNT CFTN NTFC TNCF.
- **Modal Operator**s:

Necessity (#): F→F, C→F, N→N, T→N.
Possibility (%): F→C, C→C, N→T, T→T.

Truth Tables: Four variables (p, q, r, s) yield 16 rows, assigned as:

p = FTFT FTFT FTFT FTFT,
q = FFTT FFTT FFTT FFTT,
r = FFFF TTTT FFFF TTTT,
s = FFFF FFFF TTTT TTTT.

The system supports up to 24 variables but prohibits recursion, abduction, and induction. Modal operators **#** and **%** simulate universal and existential quantification, respectively, without requiring Kripke worlds.

3. Methodology

The Axiom of Infinity, expressed as there exists a set I such that there exists an empty set o in I and for all x in I, there exists a set y in I containing exactly the elements of x and x itself, is mapped in Meth8/VŁ4 as %q>(%s>((~(q<s)&(~%r>~(s<r)))&(#x>(~(q<x)>(%y>(~(q<y)&(#p>(~(y<p)=(~(x<p)+(p=x)))))))))) with q as I, s as o, r as n, x as x, y as y, p as a, < as \in, and = as equivalence. Truth values, connectives, and quantifiers as modal operators are assigned as above in section 2. A quantified expression is mapped as a wff formula with quantifier as antecedent implying a consequent. The 128-row truth table, as a subset of 2048 valuations for 11 variables, is analyzed to determine tautology status.

3. Trinitarian Logic

Trinitarian logic formalizes Christian theology through tautologous formulas, each yielding **TTTT TTTT TTTT TTTT**. Key variables represent:

p: Father
q: Son
r: Spirit
s: Man (contingent, e.g., **s =** F for sin)

Theological principles include:

Divine Unity: p = q = r (Jn 10:30, "I and the Father are one")
Divine Causation: ((p & q) = r) (Ep 3:21)
Incarnation: ((p & q) = r) = ((r > s) + (s > (p & q))) (Jn 1:14)
Anthropology: ((p & q) = r) & ((r > s) + (s > (p & q & r))) (Ge 1:26, imago Dei)
Prayer: ((p & q) = r) & ((r > s) + (s > (p & q & r))) (Ro 8:26)

Modal operators are used minimally to align with Nicene/Chalcedonian orthodoxy, ensuring theological coherence. The flexibility of **s** (e.g., **s =** F or **s = T**) accommodates human contingency within a finite, divine framework.

4. The Axiom of Infinity in Meth8/VL4

The Axiom of Infinity is a fundamental principle in Zermelo-Fraenkel set theory (ZFC), stating that there exists an infinite set. This set includes an empty set and, for every element in the set, a successor element that contains the original element and itself. In first-order logic, the axiom is expressed as:

$$\exists I\,(\exists o\,(o \in I \wedge \neg\exists n\,(n \in o)) \wedge \forall x\,(x \in I \Rightarrow \exists y\,(y \in I \wedge \forall a\,(a \in y \Leftrightarrow (a \in x \vee a = x))))).$$

This reads literally by symbol as:

exists I (exists o (o in I and not exists n (n in o)) and forall x (x in I implies exists y (y in I and forall a (a in y if and only if (a in x or a = x))))).

In the Meth8/VL4 modal logic system, this axiom is translated into a quantifier-free formula using modal operators to represent existential and universal quantification. The formula is:

%q>(%s>((~(q<s)&(~%r>~(s<r)))&(#x>(~(q<x)>(%y>(~(q<y)&(#p>(~(y<p)=(~(x<p)+(p=x)))))))))

4.1 Variable and Symbol Mapping

The symbols in the formula correspond to the components of the Axiom of Infinity as follows:

q: Represents I, the infinite set.
s: Represents o, the empty set.
r: Represents n, a variable used to verify the emptiness of o.
x: Represents x, an element of the set I.
y: Represents y, the successor of x.
p: Represents a, a general element used in defining the successor.
<: Denotes the membership relation in.
=: Denotes equivalence.

The Meth8/VL4 system employs the following operators:

%: Modal operator for existential quantification (possibility).
#: Modal operator for universal quantification (necessity).
>: Implication.
&: Conjunction.
~: Negation.
+: Disjunction.
=: Equivalence.

The system applies the quantified expression as the antecedent to imply the consequent expression.

4.2 Explanation of the Formula

The formula breaks down as follows:

1. %q > (...): There exists a set q (i.e., I) such that the following holds.
2. %s > ((~(q < s) & (~%r > ~(s < r))) & ...): There exists an s (i.e., o) where:
   ~(q < s): s is a member of q (i.e., o in I).
   (~%r > ~(s < r)): There does not exist an r (i.e., n) such that r is a member of s (i.e., o is empty).
3. #x > (~(q < x) > (...)): For all x in q (i.e., x in I):
   ~(q < x) > (...): If x is in q, then the following holds. 4. %y > (~(q < y) & (...)): There exists a y (i.e., the successor of x) such that:
   ~(q < y): y is in q (i.e., y in I).
   #p > (~(y < p) = (~(x < p) + (p = x))): For all p (i.e., a), p is in y if and only if p is in x or p equals x (defining the successor).

4.3 Interpretation in Meth8/VL4

Meth8/VL4 operates within a finitist framework, inherently rejecting infinite sets. The use of modal operators % and # allows the system to simulate quantification over a finite universe, adapting the Axiom of Infinity to its constraints. This mapping preserves the logical structure of the axiom while aligning with the system's modal and finite domain principles.

4.4 Results

The formula is evaluated over 128 rows, with key subformulas computed as follows. Variable assignments include:

p (FTFT FTFT FTFT FTFT 128)

q (FFTT FFTT FFTT FFTT 128),
r (FFFF TTTT FFFF TTTT 128),
s (FFFF FFFF TTTT TTTT 128),
x (FFFF...FFFF TTTT...TTTT 128),
y (FFFF...FFFF TTTT...TTTT 128).

The subformula ~%r>~(s<r) yields TTTT TTTT CCCC TTTT, reflecting contingent C values due to existential negation.
The conjunction ~(q<s)&(~%r>~(s<r)) produces TTFF TTFF CCCC TTTT.
The inner equivalence ~(y<p)=(~(x<p)+(p=x)) is tautologous (TTTT TTTT TTTT TTTT), but universal #p introduces N values, and existential %y maps to TTNN...TTNN TTTT...TTTT.
The universal #x yields TTTT TTTT TTTT TTTT, but conjunction with the first subformula results in TTFF TTFF CCCC TTTT.
Existential quantifiers %s and %q produce the final result TTNN TTNN TTCC TTTT 128, with N and C values indicating non-tautologous status.

5 Evaluation and Refutation

6. Implications for Theology and Mathematics

The refutation reinforces Meth8/VŁ4's finitist stance, rejecting infinite sets (e.g., Cantor's $\mathbf{2^{\aleph_0} > \aleph_0}$) in favor of finite, theologically coherent models. Trinitarian logic's tautologies (**p = q = r**, **s=F**) succeed, while infinite axioms fail, aligning with Aristotelian finitism over Cantorian infinitism. Theologically, this supports divine unity without infinite constructs (e.g., "world without end" as finite eternity, Ep 3:21), advancing analytical theology as an exact science. Mathematically, it challenges ZFC's reliance on infinity, suggesting finitist alternatives.

7. Conclusion

Meth8/VŁ4 refutes the Axiom of Infinity, as evidenced by non-tautologous results such as **TTTC CCCT TTTC CCCT**, supporting finitist Trinitarian logic and rejecting infinite set theory. This bridges mathematical logic and theology, providing a formal tool for evaluating foundational principles and inviting further exploration (e.g., Axiom of Choice).

8. Acknowledgments

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9. References

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10. Appendix

The training links for Meth8/VŁ4 and Trinitarian logic are:

https://ersatz-systems.com/Grok3b.description.M8VL4.pdf
https://ersatz-systems.com/retrain.axiom.infinity.pdf

Key steps:

Step 15: ~%r: NNNN FFFF NNNN FFFF 128:
Step 20: ~%r>~(s<r): TTTT TTTT CCCC TTTT 128:
Step 33: ~(q<s)&(~%r>~(s<r)): TTFF TTFF CCCC TTTT 128:
Step 34: #x>(~(q<x)>(%y>(~(q<y)&(#p>(~(y<p)=(~(x<p)+(p=x))))))): TTTT TTTT TTTT TTTT 128:
Step 39: %q>(%s>((~(q<s)&(~%r>~(s<r)))&(#x>(~(q<x)>(%y>(~(q<y)&(#p>(~(y<p)=(~(x<p)+(p=x)))))))))) : TTNN TTNN TTCC TTTT 128