Many Logics, One Methodology Revisited: An Evaluation in Quad-valent Modal Logic (Meth8/VŁ4) of Christoph Benzmüller's LogiKEy Framework under Finite Quad-valent Scrutiny

A Short Response Paper

Abstract

We examine central claims of Benzmüller, Kirchner, and Pasetto (2026) — particularly their advocacy for logical pluralism via shallow embeddings in HOL (LogiKEy), the modal ultrafilter interpretation of positive properties in Gödel's ontological argument, and the derivation of uncountability under mathematical realism — through the lens of Meth8/VŁ4 (QVFL for Quad-valent Functional Logic). Meth8/VŁ4 is a quad-valent modal logic with truth values F (0,0), N (0,1), C (1,0), and T (1,1), operating over finite models via truth-table evaluation without Kripke frames. Using reductions to at most four propositional variables (p, q, r, s), we confirm support for the methodological pluralism advocated by LogiKEy. At the same time, several infinitary and modal claims are shown to be non-tautologous in this finitist setting. This illustrates both the power and the natural boundaries of pluralistic formal reasoning.

Keywords: Logical Pluralism, LogiKEy, Meth8/VŁ4, Quad-valent Logic, Gödel's Ontological Argument, Modal Ultrafilter, Finitary Metaphysics

1 Introduction

Benzmüller et al. (2026) present a case for logical pluralism at the object-logic level within a unifying classical higher-order logic (HOL) meta-framework. Their LogiKEy methodology employs shallow embeddings of higher-order modal logic (HOML) and other non-classical and non-bivalent logics into HOL. This enables transparent formalisation of complex philosophical arguments, including Gödel's modal ontological argument embellished with modal ultrafilters.

The authors contrast this pluralistic approach with "logical imperialism" — the rigid adoption of a single foundational logic in large proof assistant libraries — and argue that such rigidity impedes interdisciplinary reuse and hides foundational assumptions.

This short response paper evaluates selected key formal claims of Benzmüller et al. (2026) inside Meth8/VŁ4 (VFQL), a truth-table-driven, non-Kripkean quad-valent modal logic. A formula is considered a theorem in this system only if its complete truth table evaluates to all T (the designated *proof* value).

2 Meth8/VŁ4 as Evaluation Framework

Meth8/VŁ4 operates exclusively over finite models. It does not use Kripke frames or possible worlds semantics. Modal operators # (necessity) and % (possibility) are defined algebraically through fixed tables on the four truth values. Higher-order and modal constructions from the target paper are reduced to propositional schemata using at most four variables.

3 Evaluated Claims

3.1 Modal Ultrafilter Properties

Filter closure under conjunction:
((#(p & q) & #p) > #q)
Result: Tautologous (TTTT TTTT TTTT TTTT)

Ultrafilter maximality:
(#(p > q) > ((#p > #r) > (#q > #r)))
Result: Non-tautologous (TTCT TTTT TTCT TTTT)

3.2 HOML Embedding and Extensionality

Modalised Boolean extensionality:
(#(p = q) > (#p = #q))
Result: Tautologous (TTTT TTTT TTTT TTTT)

Single-world recovery:
(#((p = q) & (r = s)) = ((p = q) & (r = s)))
Result: Non-tautologous (NNTN TTTT TTTT NNTN)

3.3 Ontological Argument Core

Simplified God-likeness:
(#(p & q) > (r & s))
Result: Non-tautologous (TTCT TTCT TTCT TTTT)

3.4 Cardinality and Infinity Claims

Infinite entities imply infinite positives:
(((#p & #q) & #r) > #s)
Result: Non-tautologous (TTTT TTTC TTTT TTTT)

Modal Cantor-style uncountability argument:
(~(#(p = (q > r)) > #s))
Result: Non-tautologous (FNNF FNFN FFFF FFFF)

In Meth8/VŁ4's strictly finite universe, assumptions of mathematical realism cannot force uncountability of positive properties.

4 Discussion

The evaluations support the central methodological thesis of LogiKEy: different object logics and foundational assumptions can be explicitly formalised, compared, and classified within a single unifying framework. Meth8/VŁ4 realises this pluralism concretely. Classical fragments (such as modalised extensionality and basic filter closure) remain theorems, while infinitary and modal claims

become contingent or non-universal. This finitary, non-Kripkean perspective makes foundational commitments visible — particularly those depending on infinity or possible-worlds structure — thereby reinforcing Benzmüller et al.'s critique of hidden assumptions in formalised reasoning.

5 Conclusion

Benzmüller, Kirchner, and Pasetto (2026) are correct in their plea for logical pluralism in formalised reasoning. When their key claims are evaluated in Meth8/VŁ4, the pluralistic methodology is supported, while the infinitary cardinality results on positive properties are revealed to be framework-dependent rather than universally compelling. This exercise demonstrates the practical value of treating logics as negotiable parameters within a robust meta-framework. However, the authors' advocacy for HOL and HOML as the preferred unifying framework risks resembling the very logical imperialism they criticise.

Acknowledgments

All evaluations were performed in Meth8/VŁ4 (VFQL). System details and operator tables are available at https://ersatz-systems.com.

References

Benzmüller, C., Kirchner, D., & Pasetto, L. (2026). Many Logics, One Methodology: A Plea for Logical Pluralism in Formalised Reasoning. arXiv:2605.27246v1.

Appendix A: Truth Table Results

All results are based on official Meth8/VŁ4 operator definitions. Complete 16-row truth tables for every formula are available in the Meth8/VŁ4 system or upon request.