Refutation of Bourbaki's fixed point theorem and the axiom of choice

Abstract: We evaluate Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). Two versions of the theorem and then seven theorems and corollary which follow are also *not* tautologous. Therefore these conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Zarouali-Darkaoui, M. (2019). On the Bourbaki's fixed point theorem and the axiom of choice. arxiv.org/abs/1905.09782 mohssin.zarouali@gmail.com

Lemma 2 (Tarski–Bourbaki). Let E be a set, S⊂P(E), and ϕ:S→E a map such that ϕ(X)∉X for all X∈S. Therefore there is a unique subset M of E that can be well-ordered satisfying 1) for all x∈M:S_x∈S and ϕ(S_x)=x; 2) M∉S. (2.1.1)

Remark 2.1.1: We map Eq. 2.1.1 with a conjunctive consequent of 1) and 2).

LET q, r, p, s, x: E, M, ϕ, S, X

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))&~(r<s)) ;
TTTT TFTF TTTT TTTT(16)
TCTC TFTF TCTC TTTT(16)        (2.1.2)
```

Remark 2.1.3: If we map the consequent to a weakened condition of 1) implies 2), then:

```
(p=((s>q)>(((#x<x)>(x<s))>~((p&x)<x))))>(((#x<r)=(((s&x)<x)&((p&(s&x))=x)))>~(r<s)) ;
TTTT TFTF TTTT TTTT        (2.1.3)
```

Eqs. 2.1.2 or 2.1.3 are *not* tautologous, to refute Moroianu's and the Tarski-Bourbaki fixed point theorem and axiom of choice (AC). The seven theorems and corollary which follow are also *not* tautologous.

Refutation of condition/decision duality and the internal logic of extensive restriction categories

Abstract: The equations for condition/decision duality are *no*t tautologous, hence refuting what follows as internal logic of extensive restriction categories. These conjectures form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y), (x ⊑ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Robin Kaarsgaard, R. (2019). arxiv.org/pdf/1905.09181.pdf robin@di.ku.dk
Condition/decision duality and the internal logic of extensive restriction categories.

Abstract: … While categorical treatments of flowchart languages are abundant, none of them provide a treatment of this dual nature of predicates. In the present paper, we argue that extensive restriction categories are precisely categories that capture such a condition/decision duality, by means of morphisms which, coincidentally, are also called decisions. Further, we show that having these categorical decisions amounts to having an internal logic: Analogous to how subobjects of an object in a topos form a Heyting algebra, we show that decisions on an object in an extensive restriction category form a De Morgan quasilattice ...

4 The internal logic of extensive restriction categories

4.1 Kleene's three valued logics and De Morgan quasilattices

As for Boolean algebras, one can derive a partial order on De Morgan quasilattices by

p ≼ q iff p ∧ q = p, (4.1.1.1)

((p&q)=p)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.1.2)

and another one by

p ⊑ q iff p ∨ q = q. (4.1.2.1)

((p+q)=q)>~(q<p) ; TT**F**T TT**F**T TT**F**T TT**F**T (4.1.2.2)

Unlike as for Boolean algebras, however, these do not coincide, though they are anti-isomorphic, as it follows from the De Morgan laws that

$p \preccurlyeq q$ iff $\neg q \sqsubseteq \neg p$. (4.1.3.1)

((((p+q)=q)>~(q<p))>(((p&q)=p)>~(q<p)))&
~((((p&q)=p)>~(q<p))>(((p+q)=q)>~(q<p))) ;
FFFF FFFF FFFF FFFF (4.1.3.2)

Eqs. 4.1.1.2 and 4.1.2.2 are *no*t tautologous; and 4.1.3.2 is contradictory because of the *iff* in 4.1.3.1. This refutes condition/decision duality and hence what follows as internal logic of extensive restriction categories.

Refutation that Strawson's presupposition is different from Russell's entailment

Abstract: Strawson's presupposition and Russell's entailment are of the same form, equivalent, and hence not different. These conjectures form a tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ⋦ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Cohen, S.M. (2008). Strawson: "On Referring".
faculty.washington.edu/smcohen/453/StrawsonDisplay.pdf smcohen@uw.edu

The difference between entailment and presupposition:

Russell's view, that (*1*) entails (*2*), means:
(*1)* cannot be true unless (*2*) is true. If (*2*) is false, (*1*) is false. (1.1)

LET p, q: (*2*), (*1*)

(q>p)+(~q>~p) ; TTTT TTTT TTTT TTTT (1.2)

Strawson's view, that (*1*) presupposes (*2*), means:
(*1*) cannot be true or false unless (*2*) is true. If (*2*) is false, (*1*) is neither true nor false. (2.1)

(q>~(p+~p))+(~q>~(p+~p)) ; TTTT TTTT TTTT TTTT (2.2)

Remark 1: Entailment is mapped using the implication connective with the consequent and antecedent reversed in order, as it were. For example, (*1*) entails (*2*) is (*2*) implies (*1*).

Remark 2: If p equals (p or (p or not p)) and q equals (q or (q or not q), then we test if Eqs. 1.2 and 2.2 are equivalent. (3.1)

((p=(p+(p+~p)))&(q=(q+(q+~q))))>(((q>p)+(~q>~p))=((q>~(p+~p))+(~q>~(p+~p)))) ;
TTTT TTTT TTTT TTTT (3.2)

This means that Russell's and Strawson's views as rendered are of the same form, and hence entailment and presupposition are equivalents as one in the same.

Refutation of reversing the counterfactual analysis of causation

Abstract: The seminal formula of C causes E iff (~C □→ ~E) is *no*t tautologous, that is, it is *not* a theorem, from which the conjecture is derived. Hence reversing the counterfactual analysis of causation is refuted. Therefore the conjecture forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Broadbent, A. (2007). Reversing the counterfactual analysis of causation. abbroadbent@uj.ac.za www.academia.edu/attachments/1859485/download_file?st=MTU1ODQ1OTY3Miw3NS43MS4xNjEuMTQ2LDc2MDk1MzU4&s=swp-toolbar&ct=MTU1ODQ1OTY3MiwxNTU4NDU5NjgzLDc2MDk1MzU4

Abstract: The counterfactual analysis of causation has focused on one particular counterfactual conditional, taking as its starting point the suggestion that C causes E iff (~C □→ ~E). (1.1)

Remark 1.1: We interpret (~C □→ ~E) to mean □(~C → ~E).

LET p, q: C, E.

#(~p>~q)>(p>q) ; TCTT TCTT TCTT TCTT (1.2)

Eq. 1.2 as rendered is *no*t tautologous, that is, *not* a theorem, from which the conjecture is derived. Hence reversing the counterfactual analysis of causation is refuted.

Refutation of bounded and Σ_1 formulas in PA

Abstract: A fundamental proposition for bounded and Σ_1 formulas in PA is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse. Therefore the proposition is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Moschovakis, Y.N. (2014). Lecture notes in logic.
math.ucla.edu/~ynm/lectures/lnl.pdf ynm@ucla.edu

Proposition 4C.12. *Suppose T is an extension of* PA*, in the language of* PA.

(1) *The class of T-Σ_1 formulas includes all prime formulas and is closed under the positive propositional connectives & and ∨, bounded quantification of both kinds, and unbounded existential quantification.*

... to show for the proof of (1) that the class of T-Σ_1 formulas is closed under universal bounded quantification, it is enough to show that for any extended formula $\varphi(x,y,z)$,

$$T \vdash (\forall x \le y)(\exists z)\varphi(x,y,z) \leftrightarrow (\exists w)(\forall x \le y)(\exists z \le w)\varphi(x,y,z); \qquad (4.12.1)$$

LET p, w, x, y, z: *φ*, w, *x*, *y*, *z*.

(~(y>#x)&(p&((x&y)&%z)))=((~(y>#x)&~(%w<%z))&(p&((x&y)&z))) ;

```
TTTT TTTT TTTT TTTT(48),
TNTN TNTN TNTN TNTN(16),
TTTT TTTT TTTT TTTT(48)
```
(4.12.2)

the equivalence expresses an obvious fact about numbers, which can be easily proved by induction on *y*|and this induction can certainly be formalized in PA.

Eq. 4.12.2 as rendered is *not* tautologous. While the author states that the informal notes are full of errors, this fundamental mistake causes the entire section about Rosser's form of Gödel's theorems to collapse.

Refutation of logically-consistent hypothesis testing and the hexagon of oppositions

Abstract: Definitions for □H and ¬ ◊H are supposed to be equivalent for a classical mapping of agnostic hypothesis tests. While each definition reduces to a theorem in the conjecture, they are *not* tautologous. This refutes that agnostic hypothesis tests are proved to be logically consistent. Hence the characterization of credal modalities in agnostic hypothesis tests cannot be mapped to the hexagon of oppositions to explain the logical relations between these modalities. Therefore the 11 definitions tested form a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
\> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Esteves, L.G.; Izbicki, R.; Stern, J.M.; Stern, R.B. (2019).
Logically-consistent hypothesis testing and the hexagon of oppositions.
arxiv.org/pdf/1905.07662.pdf rbstern@gmail.com

Abstract: Although logical consistency is desirable in scientific research, standard statistical hypothesis tests are typically logically inconsistent. In order to address this issue, previous work introduced agnostic hypothesis tests and proved that they can be logically consistent while retaining statistical optimality properties. This paper characterizes the credal modalities in agnostic hypothesis tests and uses the hexagon of oppositions to explain the logical relations between these modalities.

Table 1: Modalities of agnostic hypothesis tests

Remark 1: We evaluate Tab. 1 beginning with Eq. 3.1 because it is the only atomic definition without the delta or nabla injections.

LET p, H; delta Δ; nabla ∇.

Modality	**Name**	**Equivalence**	**Interpretation**	
□H	Necessity (A)	ΔH∧◊H	H is accepted.	(1.1)

(#p+~(%p&~#p))&%p ;   **F**N**F**N **F**N**F**N **F**N**F**N **F**N**F**N   (1.2)

#p=((#p+~(%p&~#p))&%p) ;   TTTT TTTT TTTT TTTT   (1.3)

¬ ◊H	Impossibility (E)	ΔH∧¬ □H	H is rejected.	(2.1)

(#p+~(%p&~#p))&~#p) ; NFNF NFNF NFNF NFNF (2.2)
~%p=((#p+~(%p&~#p))&~#p) ; TTTT TTTT TTTT TTTT (2.3)

∇H Contingency (Y) ◊H∧¬ □H H is not decided. (3.1)

∇H: %p&~#p ; CCCC CCCC CCCC CCCC (3.2)

◊H Possibility (I) □H∨∇H H is not rejected. (4.1)

#p&(%p&~#p) ; FFFF FFFF FFFF FFFF (4.2)
%p=(#p&(%p&~#p)) ; NFNF NFNF NFNF NFNF (4.3)

¬ □H Non-necessity (O) ¬ ◊H∨∇H H is not accepted. (5.1)

~%p&(%p&~#p) ; FFFF FFFF FFFF FFFF (5.2)
~#p=(~%p&(%p&~#p)) ; FNFN FNFN FNFN FNFN (5.3)

ΔH Non-contingency (U) □H∨¬ ∇H H is decided. (6.1)

ΔH: #p+~(%p&~#p) ; NNNN NNNN NNNN NNNN (6.2)

Remark 1-2: Eqs. 1.3 and 2.3 as rendered result in theorems, so we test the modalities as equivalences: □H=¬ ◊H. (7.1)

#p=~%p ; CCCC CCCC CCCC CCCC (7.2)

Eq. 7.2 is *not* tautologous. This refutes that agnostic hypothesis tests are proved as logically consistent. Therefore the characterization of credal modalities in agnostic hypothesis tests cannot be mapped to the hexagon of oppositions to explain the logical relations between these modalities.

Refutation of replacing classical logic with free logic

Abstract: We evaluated 12 equations for the assertions with none tautologous. Therefore this conjecture is a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓ ,·; \ Not And;
> Imply, greater than, →, ⇒ , ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⟺, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥ , zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Bornali, P. (2019). Proposal of replacing classical logic with free logic for reasoning with non-referring names in ordinary discourse. vixra.org/pdf/1905.0358v1.pdf [no email published]

Abstract: Reasoning carried out in ordinary language, can not avoid using non-referring names if occasion arises. Semantics of classical logic does not fit well for dealing with sentences with non-referring names of the language. The principle of bivalence does not allow any third truth-value, it does not allow truth-value gap also. The outcome is an ad hoc stipulation that no names should be referentless. The aim of this paper is to evaluate how far free logic with supervaluational semantics is appropriate for dealing with the problems of non-referring names used in sentences of ordinary language, at the cost of validity of some of the classical logical theses/ principles.

3.1. Presupposition as a semantic relation … is as follows: If A and B are two propositions, then a characterization of presupposition can be given in a language as,

A presupposes B iff A is neither true nor false unless B is true. (3.1.1.1)

LET p, q: A, B. (This makes for shorter table results for propositions and not theorems.)

((q=(q=q))>~(~((p=~(p=p))+(p=~(p@p)))=(p=p)))>(p>q) ;
T**F**TT T**F**TT T**F**TT T**F**TT (3.1.1.2)

This is equivalent to, If A is true, then B is true and, If A is false, then B is true ... (3.1.2.1)

((p=(p=p))>(q=(q=q)))&((p=(p@p))>(q=(q=q))) ;
FFTT **FF**TT **FF**TT **FF**TT (3.1.2.2)

Remark 3.1.1.2-3.12.2: Eqs. 3.1.1.2 and 3.1.2.2 are *not* tautologous and *not* equivalent, as asserted, hence refuting those two conjectures.

Presupposition is different from other semantic relations, e.g., implication and necessitation.

Implication is defined as the logical truth of

'A⊃B'(~A∨B). (3.1.3.1)

(p>q)&(~p+q) ; T**F**TT T**F**TT T**F**TT T**F**TT (3.1.3.2)

For implication *modus tollens* is accepted as valid, whereas in case of presupposition it doesn't hold, since the analogue of *modus tollens* with respect to presupposition:

A presupposes B (not B) Therefore, (not A) is not valid; if both the premises are true, the conclusion is not true (i.e. neither true nor false). (3.1.4.1)

((p>q)>~q)>~p ; T**F**T**F** T**F**T**F** T**F**T**F** T**F**T**F** (3.1.4.2)

Another distinction is that the argument:

A presupposes B (not A) Therefore, B is valid in case of presupposition, since if the premises are true, so is the conclusion; whereas, for implication this argument doesn't hold. (3.1.5.1)

((p>q)>~p)>q ; **FF**TT **FF**TT **FF**TT **FF**TT (3.1.5.2)

However, presupposition and implication have something in common, which is, if A either resupposes or implies B then the argument from A to B is valid. (3.1.6.1)

Remark 3.1.6.1: We map resupposes as either Eqs. 3.1.4.1 or 3.1.5.1, or 3.1.3.1, as valid.

((((p>q)>~q)>~p)+(((p>q)>~p)>q))+((p>q)&(~p+q)) ;
T**F**TT T**F**TT T**F**TT T**F**TT (3.1.6.2)

3.3. Shortcomings of supervaluation semantics:
Considering the above case, where 'a' is denoting and 'b' is not,

'(∀x)Px⊃Pa' is true, (3.3.1.1)

LET p, q, r, s: p, x, a, b.

(r>(p=p))>((p&#q)>(p&r)) ; TTTT TTTC TTTT TTTC (3.3.1.2)

though '(∀x)Px⊃Pa' is not. (3.3.2.1)

(s>(p@p))>((p&#q)>(p&s)) ; TTTC TTTC TTTT TTTT (3.3.2.2)

However, in standard first order predicate logic (FOP) both are true as endorsed by UI rule, known as the principle of Specification. This is however quite expected in a system of free logic.

Remark 3.3: Eqs. 3.3.1.2 and 3.3.2.2 as rendered are *not* tautologous and *not* contradictory, thereby refuting four conjectures in FOP: two as true and false, and two as true.

All conjectures evaluated in 12 eqs. are *not* tautologous, and refutes replacing classical logic with free logic.

Refutation of rooted hypersequent calculus for modal logic S5

Abstract: We evaluate two example equations as *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5. The sequent calculus forms a *non* tautologous fragment of the universal logic VŁ4.

We assume the method and apparatus of Meth8/VŁ4 with Tautology as the designated proof value, **F** as contradiction, N as truthity (non-contingency), and C as falsity (contingency). The 16-valued truth table is row-major and horizontal, or repeating fragments of 128-tables, sometimes with table counts, for more variables. (See ersatz-systems.com.)

LET ~ Not, ¬ ; + Or, ∨, ∪, ⊔ ; - Not Or; & And, ∧, ∩, ⊓, ·; \ Not And;
> Imply, greater than, →, ⇒, ↦, ≻, ⊃, ↠ ; < Not Imply, less than, ∈, ≺, ⊂, ⊬, ⊭, ↞, ≲ ;
= Equivalent, ≡, :=, ⇐⇒, ↔, ≜, ≈, ≃ ; @ Not Equivalent, ≠;
% possibility, for one or some, ∃, ◊, M; # necessity, for every or all, ∀, □, L;
(z=z) T as tautology, ⊤, ordinal 3; (z@z) **F** as contradiction, Ø, Null, ⊥, zero;
(%z>#z) N as non-contingency, Δ, ordinal 1; (%z<#z) C as contingency, ∇, ordinal 2;
~(y < x) (x ≤ y), (x ⊆ y); (A=B) (A~B).
Note for clarity, we usually distribute quantifiers onto each designated variable.

From: Aghaei, M.; Mohammadi, H. (2019). Rooted hypersequent calculus for modal logic S5. arxiv.org/pdf/1905.09039.pdf aghaei@cc.iut.ac.ir, hamzeh.mohammadi@math.iut.ac.ir

Abstract: We present a rooted hypersequent calculus for modal propositional logic S5. We show that all rules of this calculus are invertible and that the rules of weakening, contraction, and cut are admissible. Soundness and completeness are established as well.

3 Rooted Hypersequent R_{S5}: Our calculus is based on finite multisets, i.e. on sets counting multiplicities of elements. We use certain categories of letters, possibly with subscripts or primed, as metavariables for certain syntactical categories (locally different conventions may be introduced) ...

Example 3.3. The following sequents are derivable in RS5. 1. ⇒(r∧p)→(q→(◊(p∧q)∧◊r)) (3.3.1.1)

(r&p)>(q>#(%(p&q)&%r)) ; TTTT TTTN TTTT TTTN (3.3.1.2)

5 Structural properties: In this section, we prove the admissibility of weakening and contraction rules, and also some properties of R_{S5}, which are used to prove the admissibility of cut rule.

5.2 Invertibility: In this subsection, first we introduce a normal form called Quasi Normal Form, which is used to prove the admissibility of the contraction and cut rules. Then we show that the structural and modal rules are invertible.

Example 5.9. ... (¬□(A→B)∨p∨◊C) ∧ (¬q) ∧ (◊A∨¬◊(A∧B)∨¬r) is in CQNF (5.9.1)

((~(#(x>y)&(p&%z))=(p=p))&~q)&((#%x+~(%(x&y)=(p=p)))+~r) ;
TT**FF** TT**FF** TT**FF** TT**FF**(64), TC**FF** TC**FF** TC**FF** TC**FF**(16),
TT**FF** TT**FF** TT**FF** TT**FF**(16), TC**FF** TC**FF** TCFF TC**FF**(32) (5.9.2)

Eqs. 3.3.1.2 and 5.9.2 as rendered are *not* tautologous, thereby refuting the rooted hypersequent calculus for modal propositional logic S5.